Learning Implicitly with Noisy Data in Linear Arithmetic

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INTRODUCTION

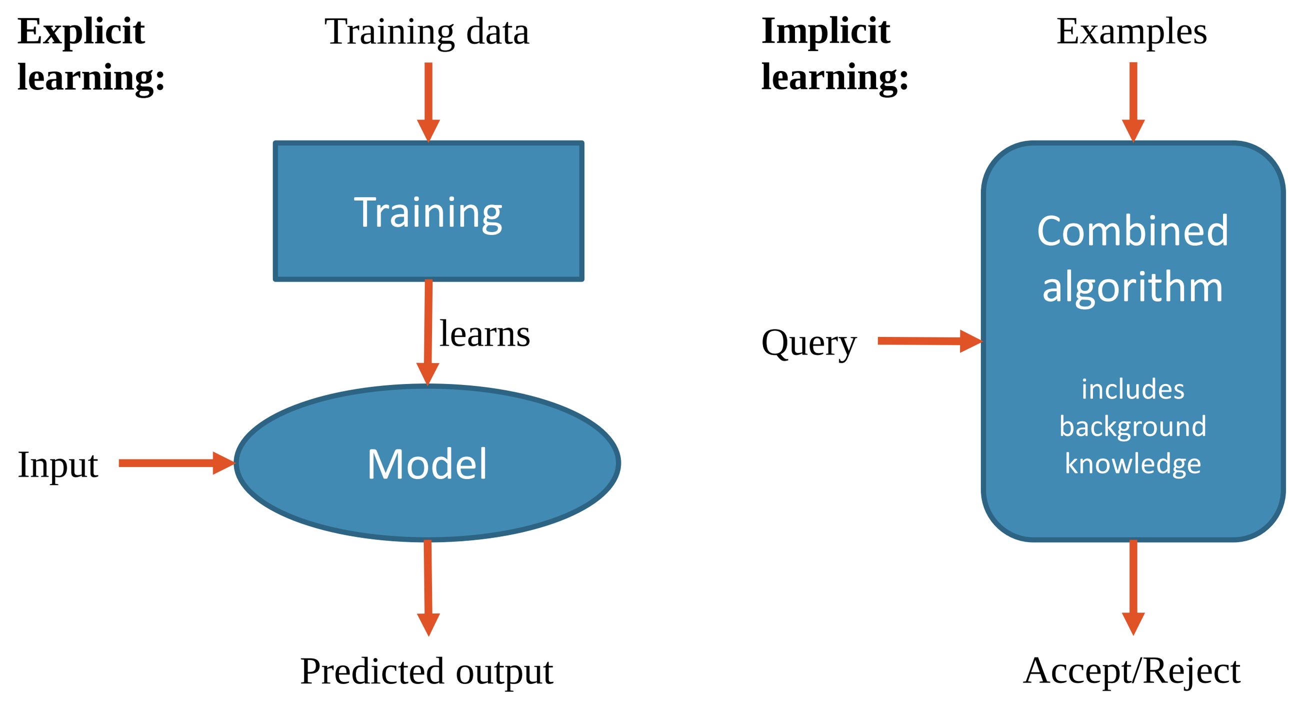
We extend an implicit learning framework to handle noisy data in the language of linear arithmetic. We prove that our extended framework keeps the existing polynomial-time complexity guarantees and provide the first empirical investigation of this hitherto purely theoretical framework.

LINEAR ARITHMETIC IN SMT

- We focus on learning in an expressive language: linear arithmetic in Satisfiability Modulo Theories (SMT)
- Quantifier-free subset of first-order logic with arithmetic operators
- E.g. $(a \geq 0) \land (b < 2a) \land (c = a + b)$
- Has polynomial-time entailment procedures

IMPLICIT LEARNING

- Learning explicit representations for SMT problems is not tractable
- Idea: Answer queries implicitly, i.e. by using examples directly
- No explicit model is created, as illustrated below



PAC-SEMANTICS

- We use the Probably Approximately Correct (PAC) Semantics framework
- Decide-PAC algorithm answers a query implicitly from examples using entailment
  - Hard-coded background knowledge Δ
  - Examples ϕ
  - Query α
  - Query accepted when $\Delta \land \phi \models \alpha$
- If entailment holds for enough examples, DecidePAC returns Accept
  - Query does not have to be fully valid, only $(1-\epsilon)$-valid. I.e. the proportion of accepted examples is at least $(1-\epsilon)$

NEW CONTRIBUTIONS

- Until now, examples had to be exact, i.e. assignments
- Idea: allow examples to be intervals, so we can handle noisy data

Theoretical contributions

- Extended the PAC-Semantics framework to accept interval-valued examples
- Proof that extended framework stays in polynomial time

Optimisation

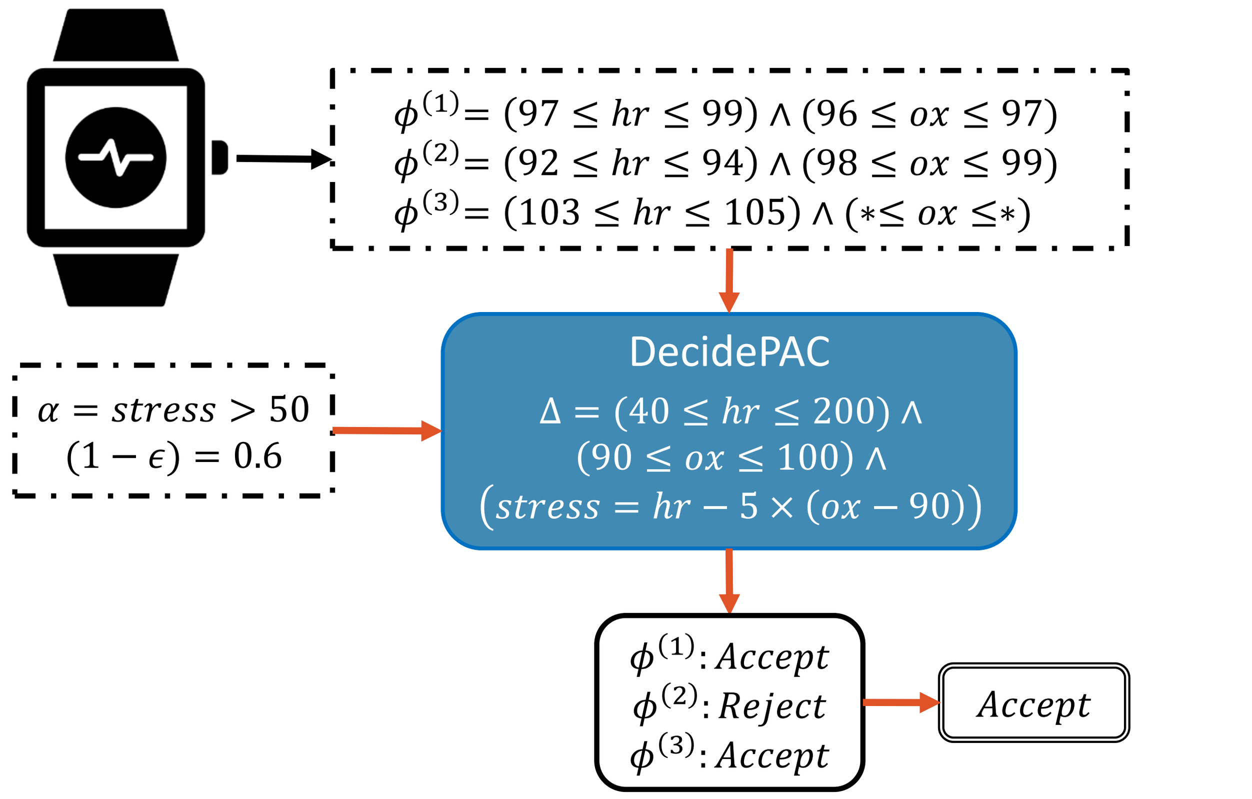
- Adapted framework to solve linear optimisation problems from examples
  - Given hard constraints, what is the optimal objective value?
- Created OptimisePAC
  - Works like exponential search
  - First we run DecidePAC repeatedly to get a rough estimate of optimal objective value
  - Then we run binary search to find optimum to desired accuracy

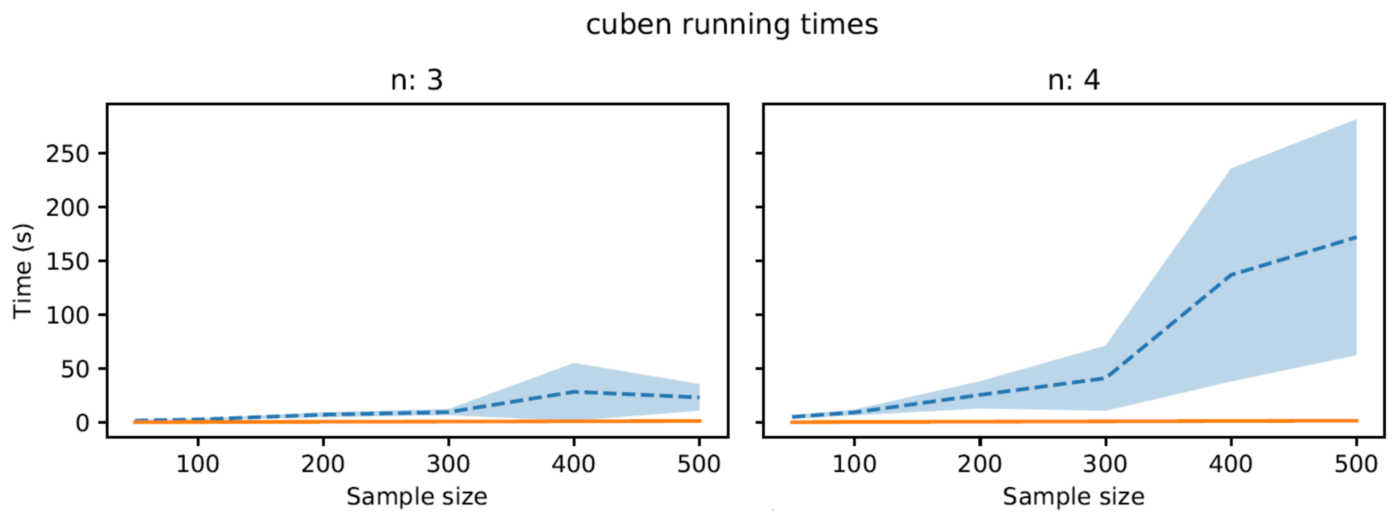
Empirical investigation

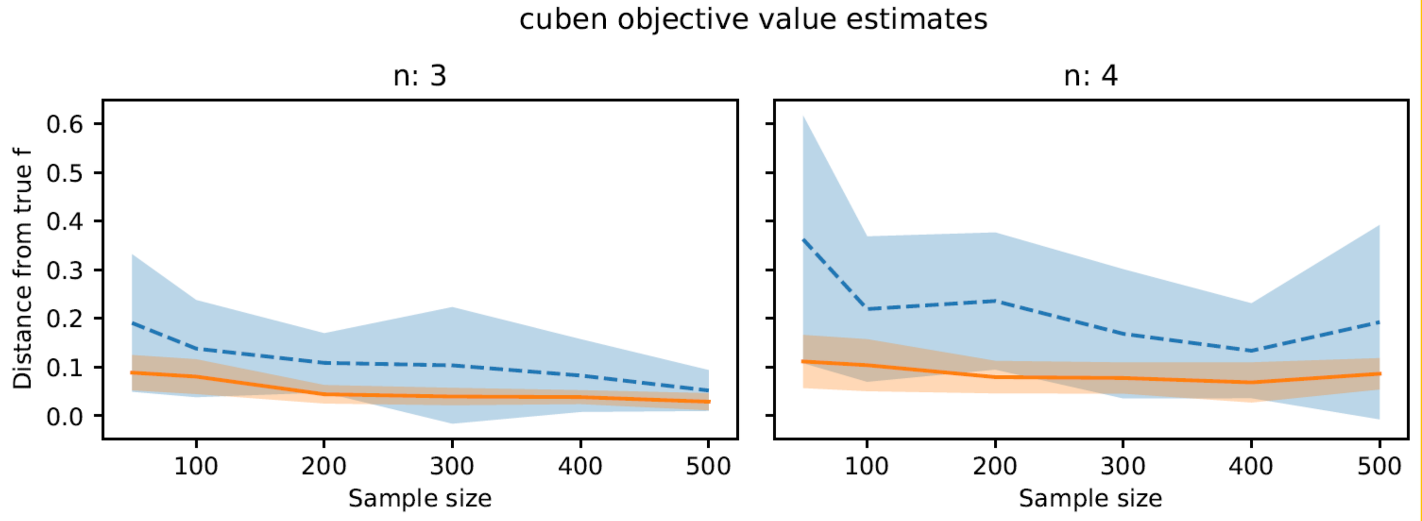
- Created first ever implementation of this framework
- Compared it with an explicit algorithm: IncaLP
  - IncaLP: Create an SMT model of the examples and then find optimum in model
  - PAC: find optimum implicitly using OptimisePAC
- Results for running time, robustness to noise and outliers are shown below

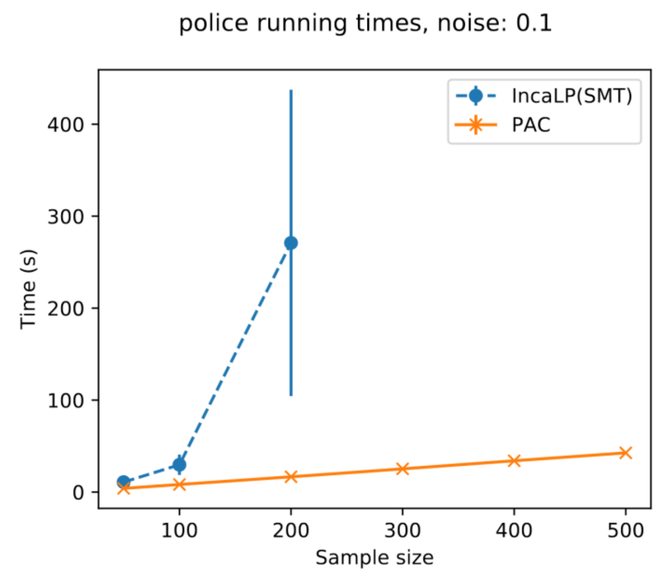
USE CASE EXAMPLE

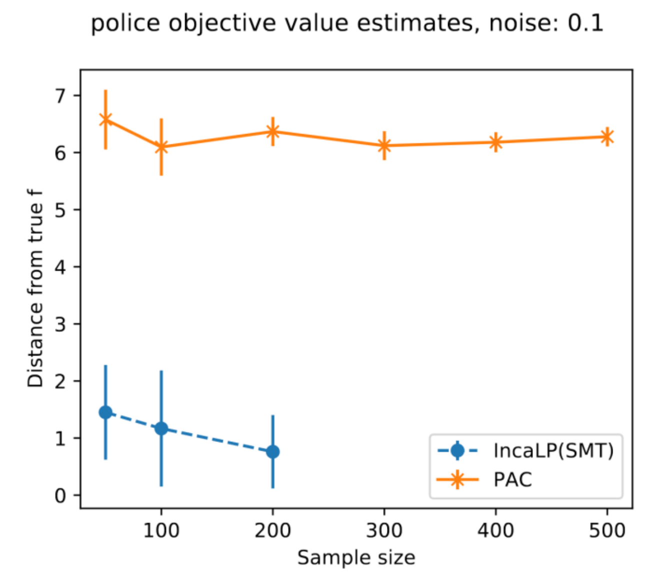
Consider a fitness watch monitoring the heart rate (hr) and blood oxygen (ox) levels of the wearer. It calculates wearer's stress level using formula: $stress = hr - 5 \cdot (ox - 90)$, which is hard-coded into its knowledge base Δ along with bounds for hr and ox. The watch alerts the user if the stress level exceeds 50, encoded as the query $\alpha = stress > 50$. The watch gets regular, but imprecise sensor readings in the form of intervals $\phi^{(k)}$. The illustration below shows that the watch answers the query using the entailment $\Delta \land \phi^{(m)} \models \alpha$ on each example, which works even when data is missing (shown as *).

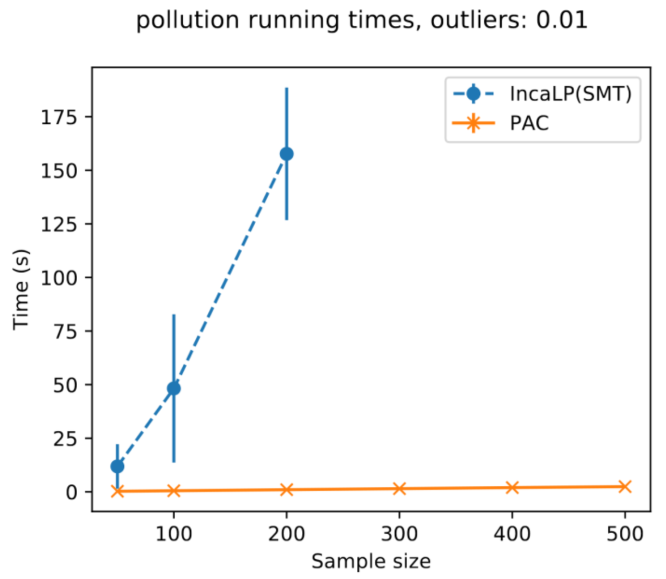


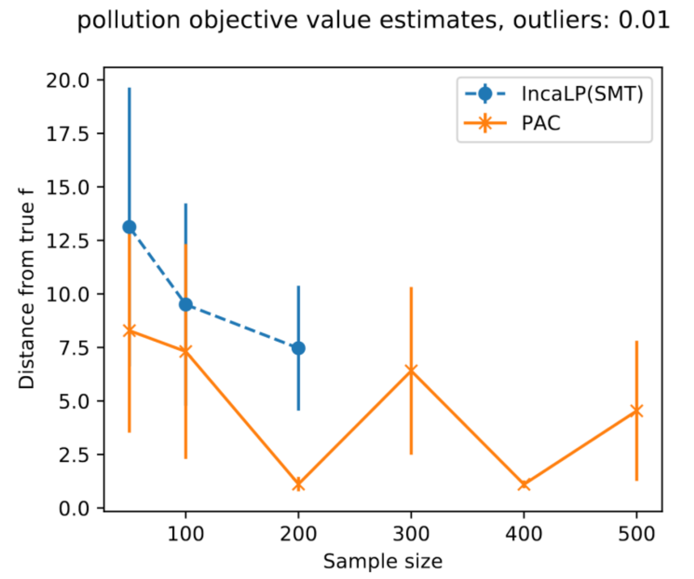












EMPIRICAL RESULTS

- For the noisy case, PAC finds similarly good estimates in significantly lower time
- Running time also grows much more slowly when increasing sample size and dimensionality
- With noise or outliers, PAC always finds an answer, while IncaLP fails to find a model in most cases
- When IncaLP finds a model, estimates can be closer to real optimum but are not always feasible
- PAC always gives feasible estimates

CONCLUSION

- We introduced ability to handle noisy data and solve optimization problem
- We have shown that skipping the step of creating an explicit model can have advantages for running time and robustness to noise and outliers
- Direction for the future: extending the framework to other classes of formulas in first-order logic and/or SMT

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